which in turn may be written in the form

$$\left[\frac{\partial}{\partial t} + (u \pm c_m)\frac{\partial}{\partial x}\right](u \pm c_m) = 0 \qquad (2.4)$$

Here c_m is the effective velocity of the sound defined by (1.2), and in the case of a conductive liquid we have $c = \sqrt{2gh}$, $v_a = b\,(gh\,/\,2\pi\rho b_0)^{1/2}$. It will be seen from system (2.4) that magnitudes $u \pm c_m$ have constant values for points moving in the conductive liquid at velocities $u \pm c_m$, i.e. for points the motions of which are defined by Eq. $dx\,/\,dt = u \pm c_m$, while the related perturbations moving towards each other do not interact between themselves.

Thus, system (2.4) coincides with the differential equations of the adiabatic flow of a perfect gas with the adiabatic exponent $k = 3$. This feature makes possible the direct application to this problem of all of the gas-dynamical results related to motions free of shock wave generation.

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STABILIZATION OF A NONLINEAR CONTROL SYSTEM IN THE CRITICAL CASE OF A PAIR OF PURELY IMAGINARY ROOTS

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We consider the problem of stabilization of the steady motions of a nonlinear control system in the critical case of a pair of purely imaginary roots. We introduce a nonanalytic control in two critical variables and use the Liapunov's classical theory of stability of motion [1 and 2] together with the methods developed in [3].

1. Let us consider the controlled system

$$\frac{dx}{dt} = Ax + Bu + g(x, u) \qquad (1.1)$$

where x denotes the $(n+2)$-dimensional perturbation vector, u is the m-dimensional control vector which we shall assume to be unaffected by any disturbances, A and B are constant $(n+2) \times (n+2)$ and $(n+2) \times m$ matrices, respectively, and $g(x, u)$

denote the terms of higher order of smallness in x and u.

If the unperturbed motion $x = 0$ of the system (1.1) does not become asymptotically stable when $u \equiv 0$, then we have a problem on stabilization, i.e. we require to find such a control $u = u(x)$ which would make, when inserted into (1.1), the unperturbed motion $x = 0$ asymptotically stable in the Liapunov sense.

Let us now consider the critical case of a pair of purely imaginary roots [2]. In this case [2 and 4] we can reduce the system (1.1), using a certain nondegenerate linear transformation of variables x_i $(i = 1, 2, \ldots, n+2)$, to

$$\frac{d\xi}{dt} = -\lambda\eta + X(\xi, \eta, z, u), \qquad \frac{d\eta}{dt} = \lambda\xi + Y(\xi, \eta, z, u) \tag{1.2}$$

$$dz / dt = A_0 z + B_0 u + a\xi + b\eta + Z(\xi, \eta, z, u) \tag{1.3}$$

where ξ and η are scalar variables, z is an n-dimensional vector with components z_s, a and b are n-dimensional constant vectors, A_0 and B_0 are $n \times n$ and $n \times m$ constant matrices, Z_s denote the components of the vector function Z and X, Y and Z_s are analytic expressions defining the nonlinearities in ξ, η, z and u.

The stabilization problem for (1.1) is equivalent to the same problem for the system (1.2) and (1.3). As we know [3], the system

$$dz / dt = A_0 z + B_0 u \tag{1.4}$$

is stabilizable and the following linear control can be constructed for it:

$$u^\circ(z) = Pz \tag{1.5}$$

A constant $m \times n$ matrix P should be chosen so, that, on substituting (1.5) into (1.4), all eigenvalues μ_s of the resulting matrix

$$C = A_0 + B_0 P = \text{const} \quad (C = (c_{ij}))$$

have negative real parts.

Let us consider the following continuous, nonanalytic control proposed by Krasovskii [4] for the system (1.2) and (1.3)

$$u(\xi, \eta, z) = Pz + w(\xi, \eta) \qquad (w = (w_1, \ldots, w_m)) \tag{1.6}$$

$$w_j(\xi, \eta) = w_j^{(1)}(\xi, \eta) + w_j^{(2)}(\xi, \eta) + \ldots + w_j^{(\omega)}(\xi, \eta) \tag{1.7}$$

$$w_j^{(k)}(\xi, \eta) = \sum_{r=-1}^{a_{jk}} \sum_{p+q-r=k} \alpha_{pq-r}^{(j)} \xi^p \eta^q \rho^{-r} \quad (k = 1, 2, \ldots \omega) \tag{1.8}$$

$$(\rho = \sqrt{\xi^2 + \eta^2};\ p \geqslant 0,\ q \geqslant 0;\ a_{jk} \geqslant 0, \omega > 0 \text{ — are all integers})$$

Functions of the form (1.8) satisfy the estimate

$$|w_j^{(k)}(\xi, \eta)| \leqslant A_j^k \|\zeta\|^k, \qquad \|\zeta\| = \sqrt{\xi^2 + \eta^2}, \qquad A_j^k = \text{const} > 0$$

which is characteristic for homogeneous, kth order forms.

Here and in the following, Expression $w_j(0, 0)$ when used in connection with a function of the type (1.8), will denote $w_j(0, 0) = \lim_{\rho \to 0} w_j(\xi, \eta) = 0$

Choice of the constants $\alpha_{pq-r}^{(j)}$ and the integers w and a_{jk} depends on the form of the initial system (1.2) and (1.3) and on, whether or not it can be stabilized.

We shall show that, transforming to the noncritical variables z_s according to

$$z_s = y_s + \varkappa_s(\xi, \eta) \tag{1.9}$$

we can reach the state when the expansions of the right-hand sides of equations for the noncritical variables will, when $y_s = 0$, contain terms depending only on the critical variables ξ and η of degree greater than $N-1$ ($N > 1$ is an integer).

We shall first define the functions $\varkappa_s(\xi, \eta)$ as the formal solution of a system of equations containing partial derivatives

$$\frac{\partial \varkappa}{\partial \xi}[-\lambda\eta + X(\xi, \eta, \varkappa, u)] + \frac{\partial \varkappa}{\partial \eta}[\lambda\xi + Y(\xi, \eta, \varkappa, u)] =$$
$$= A_0\varkappa + B_0 u + a\xi + b\eta + Z(\xi, \eta, \varkappa, u) \qquad (1.10)$$

where $\varkappa$ is a n-dimensional vector with components $\varkappa_s$.

We shall seek a solution of this system in the form of formal series

$$\varkappa_s^{(k)}(\xi, \eta) = \varkappa_s^{(1)}(\xi, \eta) + \varkappa_s^{(2)}(\xi, \eta) + \ldots \qquad (1.11)$$

where the functions $\varkappa_s^{(k)}(\xi, \eta)$ are of the type (1.8), i.e.

$$\varkappa_s^{(k)}(\xi, \eta) = \sum_{r=-1}^{b_{sk}} \sum_{p+q-r=k} a_{pq-r}^{(s)} \xi^p \eta^q \rho^{-r}, \qquad (k = 1, 2, \ldots) \qquad (1.12)$$

Inserting (1.11), (1.12) and (1.6)–(1.8) into (1.10) and comparing the νth order terms, i.e. terms for which $p + q - r = \nu$ in the left- and right-hand sides of the resulting equations, we obtain the following system of equation defining the vector function $\varkappa^\nu$

$$\lambda\left(\xi\frac{\partial \varkappa^{(\nu)}}{\partial \eta} - \eta\frac{\partial \varkappa^{(\nu)}}{\partial \xi}\right) = C\varkappa^{(\nu)} + \tau^{(\nu)}(\xi, \eta) \qquad (\nu \geqslant 1) \qquad (1.13)$$

Here $\tau_s^{(\nu)}$ denote the components of the vector function $\tau^{(\nu)}$, which are also homogeneous, νth degree functions of ξ and η of the type (1.8). When $\nu = 1$, we have $\tau^{(1)} = a\xi + b\eta + B_0 w^\nu$. When $\nu > 1$, then the functions $\tau_s^{(\nu)}$ depend on the functions $\varkappa_s^{(1)}$, $\varkappa_s^{(2)}, \ldots, \varkappa_s^{(\nu-1)}$ $(s = 1, 2, \ldots, n)$, and if $\varkappa_s^{(l)}$ where $l < \nu$ are already computed, then the functions $\tau_s^{(\nu)}(\xi, \eta)$ will be completely defined.

Let us now separate the terms appearing in the functions $\varkappa_s^{(\nu)}(\xi, \eta)$ and $\tau_s^{(\nu)}(\xi, \eta)$ which are accompanied by the factors ρ^{-r}, and write them as

$$\varkappa_s^{(\nu)} = \sum_{r=-1}^{b_{s\nu}} \varkappa_s^{(\nu+r)_\nu} \rho^{-r}, \qquad \tau_s^{(\nu)} = \sum_{r=-1}^{\sigma_{s\nu}} \tau_s^{(\nu+r)_\nu} \rho^{-r} \qquad (1.14)$$

where $\varkappa_s^{(\nu+r)_\nu}$ and $\tau_s^{(\nu+r)_\nu}$ are the $(\nu + r)$-th order forms in ξ and η, while $\sigma_{s\nu} \geqslant 0$ are integers.

Inserting (1.14) into (1.13) and assuming that

$$\xi\frac{\partial \rho^{-r}}{\partial \eta} - \eta\frac{\partial \rho^{-r}}{\partial \xi} \equiv 0 \qquad (1.15)$$

we obtain

$$\lambda\sum_{r=-1}^{b_{s\nu}} \rho^{-r}\left(\xi\frac{\partial \varkappa_s^{(\nu+r)_\nu}}{\partial \eta} - \eta\frac{\partial \varkappa_s^{(\nu+r)_\nu}}{\partial \xi}\right) = \sum_{r=-1}^{b_{s\nu}} \sum_{i=1}^{n} c_{si}\varkappa_i^{(\nu+r)_\nu}\rho^{-r} + \sum_{r=-1}^{\sigma_{s\nu}} \tau_s^{(\nu+r)_\nu}\rho^{-r} \qquad (1.16)$$

Constants $b_{s\nu}$ are chosen as follows. We put in (1.11) and (1.16)

$$b_{1\nu} = b_{2\nu} = \ldots = b_{n\nu} = \max\{\sigma_{1\nu}, \sigma_{2\nu}, \ldots, \sigma_{n\nu}\}$$

and obtain the particular values for $b_{s\nu}$ by comparing, in the left- and right-hand sides of (1.16), the terms accompanied by the same factor ρ^{-r}. In this manner we obtain for the vector function $\varkappa^{(\nu+r)_\nu}$,

$$\lambda\left(\xi\frac{\partial \varkappa^{(\nu+r)_\nu}}{\partial \eta} - \eta\frac{\partial \varkappa^{(\nu+r)_\nu}}{\partial \xi}\right) = C\varkappa^{(\nu+r)_\nu} + \tau^{(\nu+r)_\nu} \qquad (1.17)$$

The above system is a particular case of the system (32) of [1], Section 30 (see also (39.1) of [2], Section 39). By Liapunov theorem [1 and 2], the system (1.17) has a unique solution for ${\varkappa_s^{(\nu+r)}}_\nu$, which can be obtained using the method of undetermined multipliers. This will also yield the linear algebraic systems for the coefficients $a_{pq-r}^{(s)} (p + q - r = \nu)$ of the form ${\varkappa_s^{(\nu+r)}}_\nu$, which shall not be given here.

Thus Eqs. (1.17) make possible the cosecutive determination of the functions ${\varkappa_s^{(\nu+r)}}_\nu$ $(\nu = 1, 2, \ldots)$ and hence of the function $\varkappa_s^{(k)}$ of Formula (1.12).

Note 1.1. It can be shown that, if we put $\rho = \sqrt{\alpha\xi^2 + \beta\eta^2}$ (α and β are positive constants) into the control (1.6)–(1.8), then we must have $\alpha = \beta$.

Let us assume that the functions $\varkappa_s(\xi, \eta)$ (1.11) are already computed, i.e. are known up to the $(N-1)$-th order

$$\varkappa_s(\xi, \eta) = \varkappa_s^{(1)}(\xi, \eta) + \varkappa_s^{(2)}(\xi, \eta) + \ldots + \varkappa_s^{(N-1)}(\xi, \eta) \tag{1.18}$$

Inserting (1.6)–(1.8) into Eqs. (1.2) and (1.3) and transforming the result according to Formulas (1.9) and (1.18), we obtain

$$\frac{d\xi}{dt} = -\lambda\eta + \sum_{\sigma=2}^{N} X_\sigma(\xi, \eta) + \varphi_1(\xi, \eta, y), \quad \frac{d\eta}{dt} = \lambda\xi + \sum_{\sigma=2}^{N} Y_\sigma(\xi, \eta) + \varphi_2(\xi, \eta, y) \tag{1.19}$$

$$dy/dt = Cy + Z^*(\xi, \eta, y) \tag{1.20}$$

Here the functions $\varphi_i(\xi, \eta, y)$ and the components of the vector function $Z^*(\xi, \eta, y)$ do not go below the second order of smallness in ξ, η and y_s.

Functions $\varphi_i(\xi, \eta, 0)$ satisfy the Lipschitz condition with an infinitesimal constant in the estimate $|\varphi_i(\xi, \eta, 0)| \leqslant \beta_i \|\zeta\|^{N+1}$ ($\beta_i > 0$ are constants

By virtue of the transformation (1.9) and (1.18) the expansion of the components of $Z^*(\xi, \eta, 0)$ begins with the terms of the order not less than N.

If these conditions hold, then the theorem 2.2 of [5] is valid, i.e. the problem of stability of the null solution of the system (1.19), (1.20) is equivalent to the problem of stability of the null solution of

$$\frac{d\xi}{dt} = -\lambda\eta + \sum_{\sigma=2}^{N} X_\sigma(\xi, \eta), \quad \frac{d\eta}{dt} = \lambda\xi + \sum_{\sigma=2}^{N} Y_\sigma(\xi, \eta) \tag{1.21}$$

We note that the system (1.21) can also be obtained by inserting the control (1.6)–(1.8) into (1.2), replacing the components of the vector z in the resulting expression with the respective components of the vector $\varkappa$ (1.18), respectively, and terminating the result on the Nth order terms. Clearly, increasing N will not alter those of the terms of (1.19) whose initial order was not greater than N. Therefore, when performing the computations, we should first put $N = 2$ and increase it only if necessary.

2. Let us consider the homogeneous mth degree functions $u^{(m)}$ and $v^{(m)}$ of the variables ξ and η, of the type (1.8)

$$u^{(m)}(\xi, \eta) = \sum_{r=-1}^{\gamma} \sum_{p+q-r=m} c_{pq-r}\xi^p\eta^q\rho^{-r} \qquad (c_{pq-r} = \text{const}) \tag{2.1}$$

$$v^{(m)}(\xi, \eta) = \sum_{r=-1}^{\gamma} \sum_{p+q-r=m} d_{pq-r}\xi^p\eta^q\rho^{-r} \qquad (d_{pq-r} = \text{const}) \tag{2.2}$$

($\gamma \geqslant 0$, $m > 0$ are integers)

The problem which we shall now propose for the functions $u^{(m)}$ and $v^{(m)}$ will be analogous to the problem dealt with in [2]. Let the function $u^{(m)}$ be given. We require to find a function $v^{(m)}$ such, that its derivative with respect to time is, by virtue of the linear part of the system (1.21), i.e. of Eqs.

$$d\xi / dt = -\lambda\eta, \qquad d\eta / dt = \lambda\xi \tag{2.3}$$

equal to $u^{(m)}$.

The derivative of ρ^{-r} with respect to time is, by (2.3), identically equal to zero (see (1.15)). It follows therefore, that we can treat ρ^{-r} as a parameter when differentiating $v^{(m)}$ given by (2.2). Collecting all terms containing the factor ρ^{-r}, we can write (2.1) and (2.2) as

$$u^{(m)} = \sum_{r=-1}^{\gamma} u_{-r}^{(m+r)}\rho^{-r}, \qquad v^{(m)} = \sum_{r=-1}^{\gamma} v_{-r}^{(m+r)}\rho^{-r} \tag{2.4}$$

where $u_{-r}^{(m+r)}$ and $v_{-r}^{(m+r)}$ are the forms of $(m+r)$-th order in ξ and η, and where $m+r$ can be either even or odd, depending on m and r.

We shall seek the function $v^{(m)}$ (2.2) satisfying Eq.

$$\lambda\left(\xi\frac{\partial v^{(m)}}{\partial\eta} - \eta\frac{\partial v^{(m)}}{\partial\xi}\right) = u^{(m)} + \sum_{r=-1}^{\gamma} G_{-r}(\xi^2+\eta^2)^{1/2(m+r)}\rho^{-r} \tag{2.5}$$

where $u^{(m)}$ (2.1) is given and where $G_{-r} = 0$ when $m + r = 2k - 1$, $(k = 1, 2, \ldots)$.

Constants G_{-r} can be chosen so that (2.5) will have a solution. Indeed, putting (2.4) into (2.5) and comparing the terms on both sides of the resulting expression accompanied by the factor ρ^{-r}, we obtain the following equations defining the forms $v_{-r}^{(m+r)}$

$$\lambda\left(\xi\frac{\partial v_{-r}^{(m+r)}}{\partial\eta} - \eta\frac{\partial v_{-r}^{(m+r)}}{\partial\xi}\right) = u_{-r}^{(m+r)} \qquad (m+r = 2k-1) \tag{2.6}$$

$$\lambda\left(\xi\frac{\partial v_{-r}^{(m+r)}}{\partial\eta} - \eta\frac{\partial v_{-r}^{(m+r)}}{\partial\xi}\right) = u_{-r}^{(m+r)} + G_{-r}(\xi^2+\eta^2)^{1/2(m+r)} \qquad (m+r = 2k) \tag{2.7}$$

Arguments analogous to those in [2] lead to the following formula for the computation of constants:

$$G_{-r} = -\frac{1}{2\pi}\int_0^{2\pi} u_{-r}^{(m+r)}(\xi, \eta)\Big|_{\substack{\xi=\cos\theta\\ \eta=\sin\theta}} d\theta, \qquad (m+r = 2k) \tag{2.8}$$

3. We shall now investigate the stability of the reduced system (1.21), writing it as

$$d\xi / dt = -\lambda\eta + X_2(\xi, \eta) + X_3(\xi, \eta) + \ldots$$
$$d\eta / dt = \lambda\xi + Y_2(\xi, \eta) + Y_3(\xi, \eta) + \ldots \tag{3.1}$$

where X_k and Y_k represent the set of the kth order terms of the type (1.8), i.e.

$$X_k = \sum_{r=-1}^{e_{1k}} \sum_{p+q-r=k} l_{pq-r}\xi^p\eta^q\rho^{-r}, \qquad Y_k = \sum_{r=-1}^{e_{2k}} \sum_{p+q-r=k} h_{pq-r}\xi^p\eta^q\rho^{-r} \tag{3.2}$$

Here $k \geqslant 2$, $e_{1k} \geqslant 0$ and $e_{2k} \geqslant 0$ are integers. Coefficients l_{pq-r} and h_{pq-r} depend on the coefficients of the control (1.6)–(1.8); they will not, however, be given in full since they are cumbersome.

Let us consider the Liapunov function of the type

$$V = \xi^2 + \eta^2 + \varphi_3(\xi, \eta) + \varphi_4(\xi, \eta) + \ldots \tag{3.3}$$

Here the symbol $\varphi_k(\xi, \eta)$ denotes the kth order in ξ and η terms of the type (1.8).

We shall try to choose them so, that the total differential of V is, by virtue of (3.1), sign-definite, and we shall write this differential as

$$dV / dt = g_3(\xi, \eta) + g_4(\xi, \eta) + \dots \quad (3.4)$$

where $g_k(\xi, \eta)$ denote the set of the kth order terms

$$g_k(\xi, \eta) = \lambda\left(\xi \frac{\partial\varphi_k}{\partial\eta} - \eta \frac{\partial\varphi_k}{\partial\xi}\right) + \Phi^{(k)}(\xi, \eta) \quad (3.5)$$

$$\Phi^{(k)}(\xi, \eta) = 2\xi X_{k-1} + 2\eta Y_{k-1} + \sum_{i+j=k+1} \left(X_j \frac{\partial\varphi_i}{\partial\xi} + Y_j \frac{\partial\varphi_i}{\partial\eta}\right) \quad (3.6)$$

$$(k \geqslant 3,\ i \geqslant 3,\ j \geqslant 2)$$

Clearly, if the functions $\varphi_3, \dots, \varphi_{k-1}$ are known, so will be the function $\Phi^{(k)}(\xi, \eta)$. We shall turn now to the third order terms in (3.4)

$$g_3(\xi, \eta) = \lambda\left(\xi \frac{\partial\varphi_3}{\partial\eta} - \eta \frac{\partial\varphi_3}{\partial\xi}\right) + \Phi^{(3)}(\xi, \eta) \quad (3.7)$$

$$\Phi^{(3)}(\xi, \eta) = 2\xi X_2 + 2\eta Y_2 = \sum_{r=-1}^{\delta_3} \Phi_{-r}^{(3+r)}(\xi, \eta)\rho^{-r}$$

Here $\Phi_{-r}^{(3+r)}$ are the forms of the $(3 + r)$-th order in ξ and η , while $\delta_3 = \max[e_{13}, e_{23}]$. We shall choose the functions $\varphi_3(\xi, \eta)$ to satisfy

$$\lambda\left(\xi \frac{\partial\varphi_3}{\partial\eta} - \eta \frac{\partial\varphi_3}{\partial\xi}\right) = -\Phi^{(3)}(\xi, \eta) + \sum_{r=-1, 1, 3, \dots, \delta_3} G_{-r}^{(3+r)}(\xi^2 + \eta^2)^{\frac{3+r}{2}}\rho^{-r} \quad (3.8)$$

Using (2.8) to compute $G_{-r}^{(3+r)}$ we obtain

$$G_{-r}^{(3+r)} = \frac{1}{2\pi}\int_0^{2\pi} \Phi_{-r}^{(3+r)}(\xi, \eta)\Big|_{\substack{\xi=\cos\theta \\ \eta=\sin\theta}} d\theta \qquad (r = -1, 1, 3, \dots, \delta_3) \quad (3.9)$$

Putting now $G^{(m)} = \Sigma G_{-r}^{(m+r)}$ where m denotes the degree of the required homogeneous function $\varphi_m(\xi, \eta)$, we obtain the following expression for dV / dt

$$dV / dt = G^{(3)}(\xi^2 + \eta^2)\rho + \dots$$

where the terms of the order higher than third are omitted.

In the sufficiently small neighborhood of $\xi = 0$ and $\eta = 0$, the function V will be positive-definite and dV / dt will be sign definite when $G^{(3)} \neq 0$. Consequently, by the Liapunov theorem of asymptotic stability and by the first theorem on instability, the unperturbed motion of the system (3.1) will be asymptotically stable when $G^{(3)} < 0$ and unstable when $G^{(3)} > 0$. By the reduction principle (theorem 2.2 of [5]) the same assertion will be valid for the unperturbed motion of the initial system (1.2), (1.3). As the result, we have the following theorem.

Theorem 3.1. Control (1.6)–(1.8) stabilizes the system (3.1) and thus the system (1.2), (1.3), provided that the coefficients $\alpha_{pq-r}^{(j)}$ can be chosen so, that, $G^{(3)} < 0$. If $G^{(3)} > 0$ for any value of $\alpha_{pq-r}^{(j)}$, then the system (3.1) cannot be stabilized by the control (1.6)–(1.8).

We note that when $G^{(3)} \neq 0$, then only the first order terms ($p + q - r = 1$) in the control (1.6)–(1.8) need to be taken into account, since $g_3(\xi, \eta)$ is influenced only by the coefficients accompanying these terms.

Note 3.1. With a nonanalytic control (1.6)–(1.8) given, we can, in general, choose a function $\varphi_3(\xi, \eta)$ from the class of functions of the form (1.8) so, that the problem of the sign definiteness of dV/dt can be resolved by considering the set of terms of the lowest order, even if it is odd-numbered.

If $G^{(3)} = 0$ under the arbitrary choice of the coefficients $\alpha_{pq-r}^{(j)}$, then the fourth order terms in (3.4) i.e. $g_4(\xi, \eta)$ should be considered and the above procedure used for $g_3(\xi, \eta)$, repeated. This will yield $G^{(4)}$ If $G^{(4)} < 0$, then the system (3.1) can be stabilized by the control (1.6)–(1.8); if on the other hand $G^{(4)} > 0$, then stabilization is not possible.

If $G^{(4)} = 0$, we repeat the above procedure for the fifth order terms etc. If we now arrive in this manner at such m ($m \leqslant N$) that $G^{(m)} \neq 0$, then the problem on stabilization is solved as follows: if $G^{(m)} < 0$, the unperturbed motion is stabilized by the control (1.6)–(1.8), if $G^{(m)} > 0$ the stabilization is not possible.

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